Quantum corrections for $D = 4$ black holes and $D = 5$ strings

Klaus Behrndt[a]

Humboldt-Universität, Institut für Physik
Invalidenstraße 110, 10115 Berlin
Germany

Abstract

In recent times quantum corrections for $N = 2$ black holes in 4 dimensions have been addressed in the framework of double extreme black hole solutions, which are characterised by constant scalar fields. In this paper we generalize these solutions to non-constant scalar fields. This enables us to discuss quantum corrections for massless black holes and for configurations that are classically singular. We also discuss the relation to the 5-dimensional magnetic string solution.

[a]e-mail: behrndt@qft2.physik.hu-berlin.de

1. Introduction

Recently there has been a significant progress in the understanding of $N = 2$ black holes in 4 dimensions. The starting point was the observation that the central charge of a (non-singular) black hole has an extremum on the horizon and is given by the area of the horizon [1]. In addition, the horizon acts as an attractor on the scalar fields. Independently where we start at infinity the values of the scalar fields on the horizon are unique and depend only on the conserved charges. If one further assumes that the scalar fields are constant everywhere, i.e. they are frozen in their fixpoint, one has a double extreme black hole [2]. They are extreme in the sense that they saturate the BPS bound ($M = |Z|$) and their central charge (mass) is extremal (minimal). Since for these solutions the scalar fields are constant they are a good candidate for the discussion of quantum corrections. After fixing the classical starting point [3], [4] this has been done [5]. However, for these black holes it is difficult to investigate the massless case or to look for quantum solutions that are stable with less than 4 charges[b]. In [5] a less-charged solution was reachable only after inclusion of additional topological quadratic terms in the prepotential and the massless limit of double extreme black holes is the flat space time. Furthermore constant moduli are not possible for massless black holes, since they correspond to a vanishing cycle at certain point in spacetime (massless black hole singularity). Since these objects are expected to exist at points of symmetry enhancement it is desirable to have solutions that allows a massless limit. This is the case for the solution with non-constant moduli, which we are going to discuss in this paper. We will restrict ourselves on axion-free solutions.

Before we start let us fix our notation (see [5] and refs. therein). The $N = 2$ supergravity includes one gravitational, n_v vector and n_h hyper multiplets. In what follows we will neglect the hyper multiplets, assuming that these fields are constant. The bosonic $N = 2$ action is given by

$$S \sim \int d^4x \sqrt{G} \{ R - 2g_{A\bar{B}} \partial z^A \, \partial \bar{z}^B - \frac{1}{4}(\Im \mathcal{N}_{IJ} F^I \cdot F^J + \Re \mathcal{N}_{IJ} F^I \cdot {}^\star F^J)\} \qquad (1)$$

where the gauge field part $F^I \cdot F^J \equiv F^I_{\mu\nu} F^{J\,\mu\nu}$ and $I, J = 0, 1....n_v$. The complex scalar fields of the vector multiplets z^A ($A = 1..n_v$) parameterize a special Kähler manifold with the metric $g_{A\bar{B}} = \partial_A \partial_{\bar{B}} K(z, \bar{z})$, where $K(z, \bar{z})$ is the Kähler potential. Both, the gauge field coupling as well as the Kähler potential are given by the holomorphic prepotential $F(X)$

$$e^{-K} = i(\bar{X}^I F_I - X^I \bar{F}_I)$$

$$\mathcal{N}_{IJ} = \bar{F}_{IJ} + 2i\frac{(\Im F_{IL})(\Im F_{MJ})X^L X^M}{(\Im F_{MN})X^M X^N} \qquad (2)$$

with $F_I = \frac{\partial F(X)}{\partial X^I}$ and $F_{MN} = \frac{\partial^2 F(X)}{\partial X^M \partial X^N}$ (these are not gauge field components). The scalar fields z^A are defined by

$$z^A = \frac{X^A}{X^0} \qquad (3)$$

---

[b]On the classical level we need at least 4 charges to have a non-singular horizon of the black hole.

and for the prepotential we take the general cubic form

$$F(X) = \frac{d_{ABC}X^A X^B X^C}{X^0} \tag{4}$$

with general constant coefficients d_{ABC}. In type II compactification these are the classical intersection numbers of the Calabi Yau three fold. On the heterotic side these coefficients parameterize the general cubic part of the prepotential, which contains quantum corrections.

The paper is organized as follow. In the next section we describe the 4-dimensional solution. In section 3 we describe the decompactification and especially the relation to the 5-dimensional magnetic string solution. Finally, we summarize our results. In the appendix we show that the solution given in section 2 solves the equations of motion.

2. The black hole solution

In this section we are going to discuss a special solution to the Lagrangian (1). We want to restrict ourselves on axion-free solutions. This means that the scalar fields z^A and as consequence also $\mathcal{N}_{IJ}$ are pure imaginary and we will furthermore simplify the notation in setting $\Im\mathcal{N}_{IJ} \equiv \mathcal{N}_{IJ}$. Then equations of motion are

$$R_{\mu\nu} - 2\,g_{AB}(\partial_\mu z^A \partial_\nu z^B) - \tfrac{1}{2}((F \cdot F)_{\mu\nu} - \tfrac{1}{4}(F \cdot F)G_{\mu\nu}) = 0$$

$$\partial_\mu(\sqrt{G}\mathcal{N}_{IJ}F^{J\,\mu\nu})) = 0 \tag{5}$$

$$\tfrac{4}{\sqrt{G}}\partial_\mu(\sqrt{G}G^{\mu\nu}g_{A\bar{B}}\partial_\nu \bar{z}^B) - 2(\partial_A g_{B\bar{C}})\partial z^B\,\partial \bar{z}^C - \tfrac{1}{4}(\partial_A \mathcal{N}_{IJ})F^I \cdot F^J = 0$$

where $\partial_A = \frac{\partial}{\partial z^A}$.

The solution is given in terms of $n_v + 1$ harmonic functions H^A and H_0

$$ds^2 = -e^{-2U}dt^2 + e^{2U}d\vec{x}d\vec{x} \quad , \quad e^{2U} = \sqrt{H_0\,d_{ABC}H^A H^B H^C}$$

$$F^A_{mn} = \epsilon_{mnp}\partial_p H^A \quad , \quad F_{0\,0m} = \partial_m(H_0)^{-1} \quad , \quad z^A = iH_0 H^A e^{-2U} \ . \tag{6}$$

To be specific we choose for the harmonic functions

$$H^A = \sqrt{2}(h^A + \frac{p^A}{r}) \quad , \quad H_0 = \sqrt{2}(h_0 + \frac{q_0}{r}) \tag{7}$$

where h^A, h_0 are constant and related to the scalar fields at infinity. The symplectic coordinates are given by

$$X^0 = e^U \quad , \quad X^A = i\,H^A H_0 e^{-U} \ . \tag{8}$$

In comparison to the double extreme black holes [5], every charge has been replaced by a harmonic function.

Let us discuss now the charges and the mass of this solution. The electric and magnetic charges are given by the integrals over the gauge fields at spatial infinity

$$q_I = \int_{S^2_\infty} \mathcal{N}_{IJ}{}^* F^J = \int_{S^2_\infty} \mathcal{N}_{I0}{}^* F^0$$
$$p^I = \int_{S^2_\infty} F^J = \int_{S^2_\infty} F^A .$$
(9)

In the appendix we show that $\mathcal{N}_{IJ}$ is diagonal (see (24)). Thus, the black hole couples to n_v magnetic gauge fields F^A_{mn} and one electric gauge field F^0_{0m}.

To get the mass we have to look on the asymptotic geometry. First, in order to have asymptotically a Minkowski space we have the constraint $4 h_0 d_{ABC} h^A h^B h^C = 1$. Then

$$e^{-2U} = 1 - \frac{2M}{r} \pm ....$$
(10)

Thus we get for the mass

$$M = \frac{q_0}{4 h_0} + 3 p^A h_0 d_{ABC} h^B h^C .$$
(11)

Using (8) and calculating the central charge $|Z|$ we find that the black hole saturates the BPS bound

$$M^2 = |Z|^2_\infty = e^K_\infty (q_0 X^0 - p^A F_A)^2_\infty$$
(12)

where the r.h.s. has to be calculated at spatial infinity ($e^U_\infty = 1$).

On the other side if we approach the horizon all constants h^A and h_0 drop out. The area of the horizon depends only on the conserved charges q_0, p^A. Furthermore, if $q_0 d_{ABC} p^A p^B p^C > 0$ the solution behaves smooth on the horizon and we find for the area and entropy $(\mathcal{S})$

$$A = 4\mathcal{S} = 4\pi \sqrt{4 q_0 \, d_{ABC} p^A p^B p^C} .$$
(13)

If the charges and $h's$ are positive the area of the horizon defines a lower bound for the mass. Minimizing the mass with respect to h^A and h_0 gives us the area of the horizon [1]

$$4\pi M^2|_{min.} = A$$
(14)

In this case all scalar fields are constant and

$$h_0 = \frac{q_0}{c} \qquad , \qquad h^A = \frac{p^A}{c}$$
(15)

where $c^4 = 4 \, q_0 d_{ABC} p^A p^B p^C$. For these moduli all scalars are constant, i.e. coincides with their value on the horizon ($z^A \equiv z^A|_{hor.}$). By this procedure we get the double extreme black holes [2]. Taking this limit our solution (6) coincides with the solution given in [5]. There is yet another way to look on this extremization. The moduli fields are dynamical fields in $N = 2$ supersymmetric gauge theories. Especially the values at infinity (h^A, h_0) are

4

not protected by a gauge symmetry. Only the electric and magnetic charges are preserved. For a given model there is no way to fix these values. Instead one could argue that the model chooses those values for which the energy or ADM mass is minimal, i.e. the double extreme case. This is the notion of dynamical relaxation that has been introduced in [10]. Sofar we have assumed that the charges and modulis are positive. What happens if some of them are negative? There seems to be no reason to forbid negative charges. Regarded as compactification of intersecting branes it simply means that some of them are anti-branes. Immediately we come to the point of massless black holes [7]. In the $N = 4$ embedding they correspond on the type II side to a vanishing 2-cycle in the $K3$. What happens here? Let us look on a simple example with 3 vector multiplets ($A, B = 1, 2, 3$) and d_{ABC} given by

$$d_{ABC}H^A H^B H^C = H^1 H^2 H^3 + a(H^3)^3 \qquad (16)$$

with $a > 0$. On heterotic side the first term is the classical STU model and the second term corresponds to a quantum correction to this model. Let us regard this as a toy model for discussing the influence of the quantum corrections on the different types of singularities.

To get massless black holes we can take two charges negative, e.g. p^1 and p^2 (this leaves the entropy invariant). Inserting these charge into (11) we find for certain values of q_0 or h^0 massless configuration. For e^{4U} we find

$$e^{4U} = H_0 d_{ABC}H^A H^B H^C = 4(h_0 + \frac{q_0}{r}) \left((h^1 - \frac{p^1}{r})(h^2 - \frac{p^2}{r})(h^3 + \frac{p^3}{r}) + a(h^3 + \frac{p^3}{r})^3 \right) . \qquad (17)$$

The classical solution ($a = 0$) discussed in [7] ($N = 4$ embedding) was pure electric corresponding to $p^2 = p^3 = q_0 + p^1 = 0$ and $h_0 = h^A = 1$ (it defines a self-dual case) [c]. The general dyonic solution was first given in [8]. Obviously there is an additional singularity at $r = p^1$. Classically this is a naked singularity, it makes the black hole repulsive to all matter [9]. In the internal space this singularity corresponds to a vanishing cycle (see next section). If we include the quantum corrections we see that this kind of singularity (at $r = p^1 h^1$ and $r = p^2 h^2$) vanish. The quantum correction ($\sim a$) acts as a regulator, not only for the metric (or e^{4U}) but also for the prepotential and Kahler potential (see (8)). To be more concrete, for positivity the harmonic functions have to fulfill for all radii relations like

$$H^1 H^2 + b (H^3)^2 > 0 \qquad (18)$$

where $b = a$ for positivity of e^{4U}. For the Kahler metric

$$g_{A\bar{B}} = \frac{3}{4H_0} \left(-2d_{ABC}H^C + 3\frac{(d_{ACD}H^C H^D)(d_{BEF}H^E H^F)}{d_{ABC}H^A H^B H^C} \right) \qquad (19)$$

g_{33} is positive if (18) holds for $b = a - \frac{2}{3}$ and for g_{12} we have $b = -2a$ (all other components are positive without restrictions). As long as $a > \frac{2}{3}$ we find $p^1 p^2 = 2a(p^3)^2$ and h^3 has to be large enough (assuming that in (17) all constants are positive). In the case that $a < \frac{2}{3}$

---

[c]Note, on the heterotic side p^1 becomes electric (one has to go into the "stringy" basis).

we have at least for one component of the Kahler metric (e.g. g_{12}) a region in space time where it becomes negative.

What about the horizon at $r = 0$? Classically we need all 4 charges in order to have a non-singular geometry near the horizon. Also the double extreme black hole for the cubic prepotential (4) is not well defined for less than 4 charges (since the scalar fields vanish identically). For this simple quantum model, however, we see that we can set $p^1 = p^2 = 0$ and still the solution remains non-singular. It is still a Bertotti-Robertson geometry with the area of the horizon

$$A = 4\pi\sqrt{4a\,q_0(p^3)^3}\ . \tag{20}$$

and also the scalar fields are non-vanishing. On the type II side, by including of additional topological terms in the prepotential, we can even replace the electric charge q_0 [5]: $H_0 = \frac{c_2 \cdot J_3}{24}H^3$ and get a non-singular black hole solution with only one magnetic charge p^3 (c_2 is the second Chern class and J_3 is a (1,1)-form of the CY three fold).

Sofar we have regarded the model (16) as a toy model. To be realistic we have to discuss the validity of our solution. The prepotential (4) contains only the cubic terms. In general there are many other terms too. E.g. on the type II side we have neglected all instanton contributions [11] and on the heterotic side further quantum corrections [12]. Our approximation is justified as long as $|z^A| \gg 1$ and for the model (16) we have in addition the constraint that $|z^1| > |z^2| > |z^3|$ (see e.g. [5]). In all spacetime regions where these inequalities hold our solution is a good approximation. But we see already that for massless black holes or for black holes with less than 4 charges we have regions where these inequalities do not hold. E.g. the classical massless black hole singularity was given by $H_1 = 0$ and thus $z^1 = 0$, so that near this point our solution is questionable. On the other side we hope that the incorporation, e.g., of the instanton corrections do not spoil our statement that the solution behaves regular there. This, however, needs further investigation.

Since $|z^A| \gg 1$ or $|H_0| \gg 1$ is the decompactification limes to 5 dimensions let us look what the 5 dimensional solution looks like.

3. Decompactification

There are many ways to get the solution (6) by compactification of higher-dimensional configurations. On the heterotic side it is a compactification on $K3 \times T_2$. Classically our solution corresponds to the 6-dimensinal solution discussed in [6]. On the type II side we have a CY compactification, e.g. of three D-4-branes and a D-0-brane for type IIA string theory. Alternatively we can see our solution as a compactification of an intersection of 3 M-5-branes [13] and a boost along the common the string. Let us discuss the last possibility in more detail. If we have only $C_{123} = 1$ our solution (with 3 moduli $A = 1, 2, 3$) corresponds to the following intersection in 11 dimensions [14]

$$ds_{11}^2 = \frac{1}{(H^1 H^2 H^3)^{\frac{1}{3}}}\left[dudv + H_0 du^2 + H^1 H^2 H^3 d\vec{x}^2 + H^A \omega_A\right]\ . \tag{21}$$

6

This is a configuration where three 5-branes intersect over a common string and each pair of 5-branes intersect over a 3-brane. In going to 4 dimensions we first compactify over $H^A \omega_A$, with ω_A defining three 2-dimensional line elements. After this we are in 5 dimensions and have a string solution with momentum modes parametrized by H_0 (H^A are parameterizing the 5-branes). Before we generalize this solution let us look once more in the massless black hole singularity, which was given, e.g. , by $H^1 = 0$. At this point we see that one of the 2-cycles vanish ($H^1 \omega_1 = 0$). Thus, classically there is not only a singularity in the 4-dimensional spacetime but also in the internal space. Generalizing this solution to a generic CY with non-trivial intersection numbers we find for this 5-dimensional string solution

$$ds^2 = \frac{1}{(d_{ABC}H^A H^B H^C)^{\frac{1}{3}}} \left(dvdu + H_0 du^2 + (d_{ABC}H^A H^B H^C) d\vec{x}^2 \right) \ . \qquad (22)$$

Compactifying this string solution over u yields our 4-dimensional black hole solution (6). The electric gauge field is a Kalluza-Klein field, in 5 dimensions we have only magnetic gauge fields which are the same as in $D = 4$. In addition one of the 4-dimensional scalar fields is the compactification radius, which is related to $|H_0|$ and thus $|H_0| \gg 1$ gives us the decompactification limes, for which our solution is good approximation.

For the generic case ($d_{ABC}p^A p^B p^C \neq 0$) this 5-dimensional string solution is non-singular and the asymptotic geometry near the horizon is given by $AdS_3 \times S_2$.

4. Discussion

In this paper we have generalized the $N = 2$ double extreme black hole solution in [5] to the case of non-constant scalar fields (see (6)). This solution allows also a massless limit. In the classical limit the massless solution has a naked singularity where one cycle of the internal space vanishes. We argued that a cubic correction term (16) can act as an regulator for this singularity. Also we saw that for this model we can turn off 2 charges and still have a non-singular horizon. On the other side we have to take these results with care, since this model is only a good approximation for large moduli. E.g. near the massless black hole singularity this is not the case.

In a second part we have discussed the 5-dimensional magnetic string solution that yields upon compactification this black hole in 4 dimensions. This connection could be interesting with respect to the D-brane picture and microscopic state counting. The similarity to the state counting for 5-dimensional black holes in [15] is obvious. It should be possible to repeat their calculation, but now with $K3$ replaced by the CY three fold and the intersection numbers by $d_{ABC}p^A p^B p^C$. Again the electric charge is related to the momentum modes travelling along the string. We hope to come back to this point in the future.

A. The field equations

Let us convince that the solution (6) really solves the above equations (5). First, we know that if we have only $d_{ABC} = d_{123}$ it is a solution. These are the known classical black hole solutions. It is also a solution if $H^A \sim p^A f$ (where f is a harmonic function) which is the double extreme black hole. We have to discuss the case for arbitrary symmetric d_{ABC} and for general harmonic functions. In addition, for this discussion the function H_0 is irrelevant. If we are sure that it is a solution for $H_0 = 1$ we can decompactify this solution to $D = 5$ and can generate the H_0 function by a boost along the 5d (magnetic) string. (see (22)).

Gauge field equation

For this we need an expression for $\mathcal{N}$. Our prepotential and their derivative is given by

$$F = \frac{d_{ABC}X^A X^B X^C}{X^0}$$

$$F_0 = -X^0 d_{ABC}z^A z^B z^C \quad , \qquad F_A = 3X^0 d_{ABC}z^B z^C \tag{23}$$

$$F_{00} = 2 d_{ABC}z^A z^B z^C \quad , \quad F_{0A} = -3 d_{ABC}z^B z^C \quad , \quad F_{AB} = 6 d_{ABC}z^C$$

where $z^A = \frac{X^A}{X^0}$. Note, that since z^A is imaginary (axion-free) F_{0A} is real and all other second derivatives are imaginary. Inserting these terms now into (2) we find

$$\mathcal{N}_{00} = -d_{ABC}z^A z^B z^C \ , \ \mathcal{N}_{0A} = 0 \ , \ \mathcal{N}_{AB} = -6(d_{ABC}\, z^C) + 9 \frac{(d_{ADE}\, z^D z^E)(d_{BCF}\, z^C z^F)}{(d_{ABC}z^A z^B z^C)} \ . \tag{24}$$

As consequence, we have $\Re\mathcal{N} = 0$. Inserting this into the electric field equations we find

$$\partial_m \left(\sqrt{G}\mathcal{N}_{00}F^{0\,m0}\right) = \partial_m((H_0)^2 \partial_m \frac{1}{H_0}) = 0 \tag{25}$$

since H_0 is harmonic.. The magnetic field equations are solved due to the ansatz (lets ignore here the subtleties with multi-center solutions). And the Bianchi identities are solved for harmonic H^A.

Einstein equations

For Ricci tensor we have

$$R_{mn} = -\partial^2 U \, \delta_{mn} - 2\partial_m U \partial_n U \qquad , \qquad R_{00} = -\partial^2 U \, e^{-4U} \tag{26}$$

where $\partial^2 = \delta_{mn}\partial_m\partial_n$. Let us now set $H_0 = 1$. As next step we calculate the Kahler metric. Taking $K = -\log(-id_{ABC}(z - \bar{z})^A (z - \bar{z})^B (z - \bar{z})^C)$ we find $(\bar{z}^A = -z^A)$

$$g_{A\bar{B}} = \frac{1}{4}\left(6\frac{(d_{ABC}z^C)}{(d_{ABC}z^A z^B z^C)} - 9\frac{(d_{ADE}z^D z^E)(d_{BCF}z^C z^F)}{(d_{ABC}z^A z^B z^C)^2}\right) \ . \tag{27}$$

For $(F \cdot F)_{mn}$ we find

$$
\begin{aligned}
\mathcal{N}_{AB} F^A_{mp} F^B_{nq} G^{pq} &= e^{-4U} \left(\left(-\partial^2 \mathcal{D} + \tfrac{\partial \mathcal{D} \partial \mathcal{D}}{\mathcal{D}} \right) \delta_{mn} + \partial_m \partial_n \mathcal{D} - \tfrac{\partial_m \mathcal{D} \partial_n \mathcal{D}}{\mathcal{D}} - 3\, D_A \partial_m \partial_n H^A \right) \\
&= -4 \left(\partial^2 U\, \delta_{mn} - \partial_m \partial_n U \right) - 3 D_A \partial_m \partial_n H^A
\end{aligned}
$$

(28)

where we are using the compact notation $\mathcal{D} = d_{ABC} H^A H^B H^C$, $\mathcal{D}_A = d_{ABC} H^A H^B$. The scalar field part yields

$$
\begin{aligned}
g_{A\bar{B}} \partial_m z^A \partial_n \bar{z}^B &= \tfrac{1}{4} e^{-4U} \left(-\partial_m \partial_n \mathcal{D} + \tfrac{\partial_m \mathcal{D} \partial_n \mathcal{D}}{\mathcal{D}} + 3\, \mathcal{D}_A \partial_m \partial_n H^A \right) \\
&= -\partial_m \partial_n U - \partial_m U \partial_n U + \tfrac{3}{4} \mathcal{D}_A \partial_m \partial_n H^A
\end{aligned}
$$

(29)

If we now insert these expression into (5) we find that the Einstein equations are fulfilled.

Scalar field equations

This equation consists of three terms. Let us start with the first one.

$$
\begin{aligned}
\tfrac{4}{\sqrt{G}} \partial_\mu (\sqrt{G} G^{\mu\nu} g_{A\bar{B}} \partial_\nu \bar{z}^B) &= \\
&= 3i\, e^{-4U} \left(\partial^2 \mathcal{D}_A - \tfrac{1}{2} e^{-4U} \mathcal{D}_A \partial^2 \mathcal{D} - e^{-4U} \partial \mathcal{D}_A \partial \mathcal{D} + \tfrac{3}{4} e^{-8U} \mathcal{D}_A \partial \mathcal{D} \partial \mathcal{D} \right)
\end{aligned}
$$

(30)

Next we find

$$
2(\partial_A g_{B\bar{C}}) \partial z^B \partial \bar{z}^C = \frac{3i}{2} e^{-4U} \left(\partial^2 \mathcal{D}_A - e^{-4U} \mathcal{D}_A \partial^2 \mathcal{D} + \frac{1}{2} e^{-8U} \mathcal{D}_A \partial \mathcal{D} \partial \mathcal{D} \right) .
$$

(31)

Finally for the gauge field part we have

$$
\frac{1}{4} \partial_A \mathcal{N}_{BC}\, F^B \cdot F^C = \frac{3i}{2} e^{-4U} \left(\partial^2 \mathcal{D}_A - 2 e^{-4U} \partial \mathcal{D}_A \partial \mathcal{D} + e^{-8U} \mathcal{D}_A \partial \mathcal{D} \partial \mathcal{D} \right) .
$$

(32)

Putting all these expressions into (5) we find that also the scalar field equation is fulfilled.

References

[1] S. Ferrara, R. Kallosh and A. Strominger, *N = 2 exptremal black holes*, hep-th/9508072,
R. Kallosh and S. Ferrara, *Supersymmetry and attractor* , hep-th/9602136.

[2] R. Kallosh, M. Shmakova and W.K. Wong, *Freezing of moduli by N = 2 dyons*, hep-th/9607077.

[3] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova, W.K. Wong, *STU black holes and string triality*, hep-th/9608059.

[4] G. Lopes Cardoso, D. Lüst and T. Mohaupt, *Modular Symmetries of N=2 Black Holes*, hep-th/9608099.

[5] K. Behrndt, G. Lopes Cardoso, B. de Wit, R. Kallosh, D. Lüst and T. Mohaupt, *Classical and quantum N = 2 supersymmetric black holes*, hep-th/9610105.

[6] M. Cvetic and A.A. Tseytlin, *Solitonic strings and BPS saturated dyonic black holes*, hep-th/9512031.

[7] K. Behrndt, *About a class of exact string backgrounds*, hep-th/9506106.

[8] M. Cvetič and D. Youm, *Singular BPS saturated states and enhanced symmetries of four-dimensional N = 4 supersymmetric string vacua*, hep-th/9507160 ,
K.L. Chan and M. Cvetič, *Massless BPS-saturated states on the two torus moduli sub-space of heterotic string*, hep-th/9512188.

[9] R. Kallosh and A. Linde, *Exact supersymmetric massive and massless white holes*, hep-th/9507022

[10] S.-J. Rey, *Classical and quantum aspects of BPS black holes in N = 2, D = 4 heterotic string compactification*, hep-th/9610157.

[11] P. Candelas, X.C. de la Ossa, P.S. Green and L. Parkes, *A Pair of Calabi-Yau manifolds as an exactly soluble superconformal theory, Nucl. Phys.* **B359**(1991)21.

[12] J.A. Harvey and G. Moore, *Algebras, BPS states, and strings*, hep-th/9510182.

[13] G. Papadopoulos and P. Townsend, *Intersecting M-branes*, hep-th/9603087.

[14] A.A. Tseytlin, *Harmonic superposition of M-branes*, hep-th/9604035

[15] A. Strominger and C. Vafa, *Microscopic origin of the Beckenstein-Hawking entropy*, hep-th/9601029.